Active-Sterile Mixing in Big Bang Nucleosynthesis

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I. BACKGROUND

Neutrinos are some of the least well understood particles in the Standard Model. They are nearly massless ($\leq$ 2 eV) and interact only with the weak force and gravity. Given their lack of mass, and the low interaction cross-section of the weak force, they interact with other matter infrequently.

Neutrino mass eigenstates and flavor eigenstates are related by a unitary transformation. The matrix used in the unitary transformation between the flavor and mass basis is known as the Pontecorvo-Maki-Nakagawa-Sakata (MNS) matrix, and is the neutrino analogue to the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Since there are believed to be three flavors of neutrinos, the matrix can be parameterized into three angles and a CP violating phase. The angles have been experimental measured as $\sin^2 2\theta_{12} = (0.857 \pm 0.024)$, $\sin^2 2\theta_{23} > 0.95$ [2], $\sin^2 2\theta_{13} = (0.092 \pm 0.017)$ [1].

The mass squared differences between known neutrino flavors have been measured as $\Delta m^2_{21} = (7.50 \pm 0.20) * 10^{-5}$ eV^2 and $\Delta m^2_{32} = (2.32 + 0.12 - 0.08) * 10^{-3}$ eV^2 [2]. However, neither the absolute masses nor mass hierarchy ($\Delta m_{12} > \Delta m_{23}$ or $\Delta m_{12} < \Delta m_{23}$) have been experimentally determined. The CP violating phase is also unknown.

There are a number of experimental hints that the current three-flavor neutrino model is not sufficient. Both MiniBooNE and LSND detected an excess of low-energy events that would indicate a higher rate of $\nu_\mu \rightarrow \nu_e$ appearance than expected [3]. Furthermore, several reactor-based experiments (Gosgen, SRP, ILL, Bugey, etc.) have seen fewer than expected antineutrino [10]. Gallium-based solar neutrino experiments (GALLEX and SAGE) have also seen several percent fewer antineutrino events than expected [6].

All of these suggest that there might be a fourth type of neutrino. All currently known neutrinos are right-handed; no left-handed neutrinos have ever detected, nor any right-handed antineutrinos. It is possible that left-handed neutrinos (and right-handed antineutrinos) are "sterile" – that is, they do not interact with the weak force. Such particles would be nearly undetectable, given the already-low interaction cross-section of active neutrinos.

While unlikely to interact directly, sterile neutrinos would affect the interactions of active neutrinos. As neutrinos oscillate from electron to mu to tau flavors, they could also oscillate to a sterile state.

This could impact the formation of matter, as the neutrino flux during big bang nucleosynthesis (BBN) fixes the proton-to-neutron ratio. BBN occurs when the universe's plasma temperature drops to about 1 MeV. At that point, the universe has cooled enough so that protons and neutrons can enter bound states.

At temperatures above 0.7 MeV, the weak interaction rate is fast in comparison to the universe's expansion rate. The forward and reverse reactions of proton-to-neutron conversions are approximately equal.

$$\mathrm{n} + \mathrm{e}^+ \rightarrow \bar{\nu}_\mathrm{e} + \mathrm{p} \tag{1}$$

$$\mathrm{n} + \nu_\mathrm{e} \rightarrow \mathrm{p} + \mathrm{e}^- \tag{2}$$

As the above reactions slow, the flux of available electron neutrinos fixes the neutron-to-proton ratio. By weak freeze-out, the second reaction occurs at a much slower rate than the first and the proton-to-neutron ratio is approximately seven-to-one.

The vast majority of the neutrons are incorporated into alpha particles (helium nuclei), although about $\frac{1}{10{,}000}$ neutrons are incorporated into deuterium nuclei. Very small amounts of tritium, lithium-6, lithium-7, lithium-8 and beryllium-7 are also produced during BBN.

Given that neutrinos oscillate between flavors, the flux from *all* flavors impacts the electron neutrino flux. Therefore, oscillations between electron neutrinos and sterile neutrinos can affect the neutron-to-proton ratio and the primordial elemental abundances.

The predicted helium-4 and deuterium abundances have matched observation almost exactly. Lithium has been less well-predicted, but it is possible that current theories of the role of Li^7 in stellar development is at fault, rather than standard BBN [12].

The addition of a sterile neutrino – fully thermalized or not – could significantly change elemental abundances. If the sterile neutrinos decoupled before BBN, they could carry significant radiation energy density with them.

Current cosmological measurements do not rule out such neutrinos – Planck's 2013 results reported N_{eff}, the "effective number of degrees of freedom" or the effective number of neutrinos as 3.30 $\pm$ 0.27 and the sum of the light neutrino masses as 0.27 eV [13]. The standard model of cosmology would predict $N_{eff} = 3.05$, but sterile neutrinos could also contribute fractional amounts, as follows:

The active neutrinos decouple at "weak freeze out", at which point the proton-neutron ratio is fixed. At this point, further entropy from electron-positron annihilations is transferred only to photons, not to both neutrinos and photons. Therefore, the photon temperature is slightly higher than the neutrino temperature:

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{\frac{1}{3}} \tag{3}$$

From this, we can determine the relativistic energy density in terms of the photon energy density (measurable from the CMB):

$$\rho_r = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{1}{3}} N_{eff}\right]\rho_\gamma \tag{4}$$

Finite temperature QED corrections and incomplete decoupling of the active flavors result in a theoretical N_{eff} value of 3.045 [9].

If there existed sterile neutrinos decoupled at a much higher temperature than left-handed neutrinos, but had a given a non-zero active-sterile mixing angle, the thermally-decoupled sterile neutrinos would affect thermally-coupled active neutrinos just before weak freeze-out. Such neutrinos would add only fractional amounts to N_{eff}, resulting in a value within current experimental bounds.

II. METHODOLOGY

For simplicity, we simulated a system in which only electron neutrinos and sterile neutrinos oscillate – $\nu_\mu \rightarrow \nu_s$ and $\nu_\tau \rightarrow \nu_s$ oscillations are not included.

Therefore, a sterile neutrino of flavor can be described as:

$$|\nu_s\rangle = \sum_k U^*_{sk}|\nu_k\rangle \tag{5}$$

with vacuum Hamiltonian and Schrödinger-like equation:

$$\hat{H}_{vac} = p + \frac{m^2}{2p} \tag{6}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle \tag{7}$$

The wavefunction of the sterile neutrino can be written in either flavor or mass basis. We assume that the addition of a sterile neutrino also introduces a fourth mass eigenstate. In our simplified system, we only use two mass eigenstates – corresponding to m_1 and m_4 in a four-flavor system.

$$\psi_f = \begin{bmatrix} \langle\nu_e|\psi\rangle \\ \langle\nu_s|\psi\rangle \end{bmatrix} \tag{8}$$

$$\psi_m = \begin{bmatrix} \langle\nu_1|\psi\rangle \\ \langle\nu_2|\psi\rangle \end{bmatrix} \tag{9}$$

$$\psi_f = U_m\psi_m \tag{10}$$

Therefore, the equations for the propagation of neutrinos of a given flavor become:

$$|\nu_e\rangle = \cos\theta(t)|\nu_1(t)\rangle + \sin\theta(t)|\nu_2(t)\rangle \tag{11}$$

$$|\nu_s\rangle = -\sin\theta(t)|\nu_1(t)\rangle + \cos\theta(t)|\nu_2(t)\rangle \tag{12}$$

In vacuum, this can be written as:

$$\begin{aligned} i\frac{\partial}{\partial t}\psi_f \quad &= \Bigg(\left(p + \tfrac{m_1^2+m_2^2}{4p}\right)\hat{I} \\ &\quad + \tfrac{1}{2}\tfrac{\delta m^2}{2p}\begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}\Bigg) \end{aligned} \tag{13}$$

The energy can be written in terms of temperature – that is, $E = k_BT$. In natural units, $k_B = 1$, so:

$$P_{e\rightarrow s} = \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 L}{4T}\right) \tag{14}$$

We assume that at BBN, electron neutrinos are ultra-relativistic and travel at the speed of light, and as such the distance traveled is equal to the time * the speed of light. In natural units, c is also equal to 1, so we rewrite:

$$P_{e\rightarrow s} = \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 t}{4T}\right) \tag{15}$$

To apply the Runga-Kutta method, we use the time derivative of this probability to estimate the change in probability in a given unit of time.

$$\dot{P} = 2\sin^2\theta\sin\left(\frac{\delta m^2 t}{4T}\right)\cos\left(\frac{\delta m^2 t}{4T}\right) \tag{16}$$

As a first approximation, we assume that the neutrino oscillations are similar to those in vacuum. In future, we intend to add matter-enhanced oscillations appropriate to the density of the universe at BBN. Due to increasingly complexity, we do not at this time intend to implement oscillations between more than two flavors.

We are using the NUC123 code, with default values for model and computation parameters [7]. It is based on the work of [15] and subsequent revisions such as Wagoner [14], with care taken towards Fortran 77's unusual memory management.

From the output of the code, we can then determine if BBN is consistent with a sterile neutrino species that decoupled much earlier than weak freeze-out.

The code has an arbitrary number of sterile neutrino species, which have independently-defined temperatures. This allows us to simulate both fully thermalized sterile neutrinos and non-fully-thermalized sterile neutrinos.

While this is not a precise calculation of oscillations in a multi-sterile system, it does allow us to easily simulate systems in which there is more than one sterile neutrino, and the two (or more) steriles did not decouple at the same time.

Furthermore, it allows us to simulate systems with different mixing angles without complex algebra; a sterile with sufficiently large mixing angle will interact with actives enough to be in thermal equilibrium with the actives, while a sterile with a smaller mixing angle will be at a lower temperature.

III. PRELIMINARY RESULTS

A minimal model to explain the LSND and MiniBooNe anomalies was presented at INFO13 by Cecilia Lunardini [8]. She favored a 3+2 model with one sterile neutrino fully thermalized (mixing angle up to 0.3) and the other less so [4]. Both steriles would be very light – $\Delta m_4 = 0.5$ eV, $\Delta m_5 = 0.9$.

Irina Mocioiu advanced a similar model, although with slightly lower mixing angles: $|U_{e4} = 0.149$, $|U_{e5}| = 0.127$, $|U_{\mu 4}| = 0.112$, $|U_{\mu 5}| = 0.127$ [11].

We simulated Dr. Lunardini's model with two fully-thermalized sterile neutrinos, and Dr. Mocioiu's model with two steriles at 40% the temperature of the actives. Dr. Lunardini also mentioned the possibility of a 3+2 model where one sterile was fully thermalized, and the other was not – again, we used 40% active neutrino temperature as a first approximation. We also simulated a models in which there was only one sterile neutrino, both fully thermalized and at 40% the active neutrino temperature.

Note that these calculations do not include oscillations; rather, we assume that oscillations will result in differing levels of thermalization.

	D/H	Y_p ($*\ 10^{-5}$)
measurement ([5], [2])	(2.82±0.21)	(0.2565 ± 0.0060)
3+1 thermalized	2.972	0.2582
3+2 thermalized	3.320	0.2691
3+1, $T_{\nu_s} = 0.4T_\nu$	2.631	0.2461
3+2, 1 $T_{\nu_s} = 0.4T_\nu$	2.981	0.2585
3+2, both $T_{\nu_s} = 0.4T_\nu$	2.640	0.2464

The 3+2 fully thermalized model is excluded because both D/H and Y_p are outside current measurements. The 3+1 fully thermalized model results in a D/H value very close to the error bound; better measurements of primordial (D/H) may allow us to exclude this model as well.

David Tytler (UCSD) should soon be able to measure Y_p to 2%. With this precision, we should be able to evaluate if the 3+2 model where only one sterile is fully thermalized is consistent with observation.

However, it is also clear there is a great deal of parameter space for sterile neutrinos consistent with BBN. However, the mixing angles must be small – fully thermalized sterile neutrinos cause deuterium to be over-produced relative to our observations. In other words, this data suggests that all sterile neutrinos must decouple earlier than the active neutrinos, at some point before BBN.

IV. FURTHER WORK

This project will be continued as a senior thesis under Dr. Christian Ott (Caltech), working in collaboration with Dr. George Fuller (UCSD). We intend to run the above routine over a much wider range of sterile neutrino temperatures and masses, simulating both 3+1 and 3+2 models.

We also intend to add calculations of cosmological parameters (N_{eff}, $\sum m_\nu$). We cannot simply use eqn. 4 – if the sterile neutrinos decouple before weak freeze-out, they are no longer ultrarelativistic at photon decoupling. We also cannot treat them as non-relativistic matter, as their energies are still high enough to warrant relativistic corrections.

Evan Grohls (UCSD) has been working on this problem. We wish to compare our values against experimentally determined values from Planck. However, Planck does not measure N_{eff} directly; rather, it measures the the ratio between the sound horizon and the diffusion length.

We first focus on the sound horizon. N_{eff} can be written in terms of the scale factors at the current epoch and at matter-radiation equality.

$$\frac{a_{eq}}{a_0} = \frac{\left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right) 8\pi^3 G T_{\gamma,0}^4}{45 H_0^2 \Omega_m} \tag{17}$$

$$a_{eq} = \frac{1 + 0.2271 N_{eff}}{4.041 * 10^4 \Omega_m h^2} \tag{18}$$

From this, we can calculate the Hubble rate and radius. Once we have determined the Hubble radius, we must consider the sound horizon. In terms of the conformal distance, the sound horizon is:

$$r_s = \int_0^\eta c_s d\eta' \tag{19}$$

Planck does not measure conformal distance, but rather the proper distance (the angle subtended on the sky), so the proper sound horizon is:

$$d_s = a_* \int_0^{\eta_*} c_s d\eta' = a_* r_s \tag{20}$$

The speed of sound is defined as:

$$c_s^2 = \frac{\partial P}{\partial \rho} \tag{21}$$

Before photon decoupling (the creation of the CMB), photons and baryons exclusively interact with each other, and the sound speed can be written as:

$$c_s^2 = \frac{\partial P}{\partial \rho_\gamma}\frac{d\rho_\gamma}{d\rho} + \frac{\partial P}{\partial \rho_b}\frac{d\rho_b}{d\rho} \tag{22}$$

From this, we can determine $\frac{d\rho_b}{d\rho_\gamma}$ (which we will now call R), and apply the radiation equation of state.

$$c_s = \frac{1}{\sqrt{3(1+R)}} \tag{23}$$

This can be rewritten in terms of the scale factor, a, and therefore, can also be written in terms of a_{eq}. We define R at matter-radiation equality as R_{eq} and parameterize in terms of N_eff.

$$R_{eq} = \frac{3}{4}\frac{\Omega_b}{\Omega_m}(1 + 0.2271N_{eff}) \tag{24}$$

We define $k_{eq} = a_{eq}H(a_{eq})$ and $R_* = \mathrm{R}$ in terms of co-moving distance. When solved through, the sound horizon becomes:

$$r_s = \frac{2}{3k_{eq}}\sqrt{\frac{6}{R_{eq}}}\ln\left(\frac{\sqrt{R_{eq}+R_*}+\sqrt{1+R_*}}{1+\sqrt{R_{eq}}}\right) \tag{25}$$

The co-moving diffusion length can be written in terms of the mean free path (λ_{map}). We assume that photons diffuse through the electron plasma by Thompson scattering.

$$r_d = \sqrt{N}\frac{\lambda_{mfp}}{a} \tag{26}$$

$$d(r_d)^2 = \frac{\lambda_{mfp}^2}{a^2}dN = \frac{d\eta}{n_e\sigma_t a} = \int_0^{\eta_*}\frac{d\eta}{n_e\sigma_T}a \tag{27}$$

Written in terms of damping wavenumber, the expression is:

$$k_d^{-2} = \frac{4}{27}\int_0^{\eta}\frac{d\eta'}{n_e\sigma_T a} = \frac{4}{27}\int_0^{a_*}\frac{da}{n_e\sigma_t a^3 H} \tag{28}$$

The Hubble rate is the same as above:

$$H^2 = \frac{8\pi G}{3}\left(\frac{a_{eq}}{a}\right)^3\rho_{m,eq}\left(1+\frac{a}{a_eq}\right) \tag{29}$$

We can (with some algebra) rewrite the damping wavenumber as:

$$k_d^{-2} = \frac{8}{135}\frac{a_*^{5/2}}{a_{eq}^{3/2}n_{e,0}\sigma_T}\sqrt{\frac{3}{8\pi G\rho_{m,eq}}} \left(\left(1+\frac{a_{eq}}{a_*}\right)^{5/2} - \frac{10}{3}\frac{a_{eq}}{a_*}\left(1+\frac{a_{eq}}{a_*}\right)^{3/2} +5\left(\frac{a_{eq}}{a_*}\right)^2\left(1+\frac{a_{eq}}{a_*}\right)^{1/2} - \frac{8}{3}\left(\frac{a_{eq}}{a_*}\right)^{5/2}\right) \tag{30}$$

To solve for the damping wavenumber, we rewrite the electron number density as:

$$n_e = \frac{\rho_b}{m_b}\left(1-\frac{Y_p}{2}\right) \tag{31}$$

$$n_{e,0} = \frac{3\Omega_b H_0^2}{8\pi m_b G}\left(1-\frac{Y_p}{2}\right) \tag{32}$$

We also write $\rho_{m,eq}$ in terms of a_{eq}:

$$\rho_{m,eq} = \frac{3\Omega_m H_0^2}{8\pi G a_{eq}^3} \tag{33}$$

Substituting these in and solving (with H_0 = h * 10^2 km/s/Mpc), we determine:

$$k_d^{-2} = 8.381 * 10^4\ \mathrm{Mpc}^2 a^{5/2}(\Omega_m h^2)^{-1/2} (\Omega_b h^2)^{-1}\left(1-\frac{Y_p}{2}\right)^{-1}\left(\left(1+\frac{a_{eq}}{a_*}\right)^{5/2} -\frac{10}{3}\frac{a_{eq}}{a_*}\left(1+\frac{a_{eq}}{a_*}\right)^{3/2} + 5\left(\frac{a_{eq}}{a_*}\right)^2\left(1+\frac{a_{eq}}{a_*}\right)^{1/2} -\frac{8}{3}\left(\frac{a_{eq}}{a_*}\right)^{5/2}\right) \tag{34}$$

We take $k_d = 2\pi/r_d$, and the proper diffusion length as $d_d = ar_d$. Therefore, the diffusion length is:

$$d_d = 7.49 * 10^{-5}\ \mathrm{Mpc}^2 a^{7/2} \left((\Omega_m h^2)^{-1/2}(\Omega_b h^2)^{-1}\left(1-\frac{Y_p}{2}\right)^{-1} \left(\left(1+\frac{a_{eq}}{a_*}\right)^{5/2} - \frac{10}{3}\frac{a_{eq}}{a_*}\left(1+\frac{a_{eq}}{a_*}\right)^{3/2} +5\left(\frac{a_{eq}}{a_*}\right)^2\left(1+\frac{a_{eq}}{a_*}\right)^{1/2} - \frac{8}{3}\left(\frac{a_{eq}}{a_*}\right)^{5/2}\right)\right)^{-1} \tag{35}$$

None of the above has yet been implemented into our Fortran code. This is the obvious next step: we must be able to determine which models fall within the accepted range for cosmological parameters as well as elemental abundances.

CMB measurements also measure the so-called "sum of the light neutrino masses". We have not yet determined how this quantity scales with energy density, temperature, and time. We plan to discuss further with collaborators on Planck in order to correctly compare our models to their $\sum m_\nu$ measurements.

We would like to plot contours of currently experimentally accepted values on a plot of N_{eff} vs. sterile neutrino mass. We will also include contours based on the increased precision from future measurements, allowing us to determine which models can be excluded from the parameter space in the near future.

Once we have determined what parameters are physically allowable, we will prepare a manuscript for publication.

[1] An, F. P., Bai, J. Z., Balantekin, A. B., and et al.: 2012, *Phys. Rev. Lett.* **108**, 171803

[2] Beringer, J. and et al.: 2012, *Phys. Rev. D* **86**, 010001

[3] Dib, C., Helo, J. C., Kovalenko, S., and Schmidt, I.: 2011, *Phys. Rev. D* **84(7)**, 071301

[4] Giunti, C. and Laveder, M.: 2011, *PRD* **84(9)**, 093006

[5] Izotov, Y. I. and Thuan, T. X.: 2010, *ApJ Letters* **710**, L67

[6] Kopp, J., Machado, P. A. N., Maltoni, M., and Schwetz, T.: 2013, *Journal of High Energy Physics* **5**, 50

[7] Lawrence Kawano: 1992, *Let's Go: Early Universe: Guide to Primordial Nucleosynthesis Programming*

[8] Lunardini, C.: 2013, *Light sterile neutrinos in the early universe*, INFO13

[9] Mangano, G., Miele, G., Pastor, S., and Peloso, M.: 2002, *Physics Letters B* **534**, 8

[10] Mention, G., Fechner, M., Lasserre, T., Mueller, T. A., Lhuillier, D., Cribier, M., and Letourneau, A.: 2011, *Phys. Rev. D* **83(7)**, 073006

[11] Mocioiu, I.: 2013, *Exploring sterile neutrinos with Ice-Cube and Deep Core*, INFO13

[12] Olive, K. A. and Schramm, D. N.: 1992, *Nature (London)* **360**, 439

[13] Planck Collaboration, Ade, P. A. R., Aghanim, N., Armitage-Caplan, C., Arnaud, M., Ashdown, M., Atrio-Barandela, F., Aumont, J., Baccigalupi, C., Banday, A. J., and et al.: 2013, *ArXiv e-prints*

[14] Wagoner, R. V.: 1973, *Astrophys. J.* **179**, 343

[15] Wagoner, R. V., Fowler, W. A., and Hoyle, F.: 1967, *Astrophys. J.* **148**, 3